Minnesota Comprehensive Assessments (MCA)

DRAFT Mathematics

Test Specifications

for MCA-III, Grade 11

and MCA-Modified, Grade 11

Minnesota Department of Education

January 23, 2014

Based on the
Minnesota K-12 Academic Standards in Mathematics (2007 version)

MINNESOTA DEPARTMENT OF EDUCATION

DRAFT Mathematics
Test Specifications
for MCA-III, Grade 11
and MCA-Modified, Grade 11

An updated version of this document will be published annually, typically in August, until the 2014 operational administration of the test.

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TABLE OF CONTENTS

MCA MATHEMATICS TEST SPECIFICATIONS 5
INTRODUCTION 5
PURPOSE OF THE MINNESOTA COMPREHENSIVE ASSESSMENTS 5
MCA-III 5
MCA-Modified 6
PURPOSE AND OVERVIEW OF THE TEST SPECIFICATIONS 7
ITEM SPECIFICATIONS 8
Item Specification Considerations 8
COGNITIVE COMPLEXITY 10
CALCULATOR DESIGNATION 12
TEST DESIGN BY GRADE LEVEL 12
A GUIDE TO READING THE TEST SPECIFICATIONS 15
AN EXPLANATION OF TERMS RELATED TO THE GRADE-LEVEL TABLES 16
MATHEMATICS MCA-III TEST SPECIFICATIONS GRADE 11 LEVEL TABLES 17

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MCA MATHEMATICS TEST SPECIFICATIONS

Introduction

Mathematics test specifications for grade 11 of the Minnesota Comprehensive Assessments Series III (MCA-III) and grade 11 of the MCA-Modified are presented in this document. The reader is encouraged to read the introductory information carefully because many important concepts are presented, including the purposes of the MCA-III and MCA-Modified in Mathematics, a description of the cognitive levels and other information about the format of the test specifications.

Purpose of the Minnesota Comprehensive Assessments

The purpose of Minnesota assessments is to measure Minnesota students' achievement with regard to the Minnesota Academic Standards. Assessment results can be used to inform curriculum decisions at the district and school level, inform instruction at the classroom level and demonstrate student academic progress from year to year.

MCA-III

The passage of the No Child Left Behind Act (2000)[1] required that students be assessed in grades 3–8 and high school. The *Minnesota K–12 Academic Standards in Mathematics* were adopted in 2003; the Minnesota Comprehensive Assessments-Series II assessed these standards. The 2006 Minnesota Legislature approved the 2006 Omnibus Education Policy Act (see Minn. Stat. § 120B.023, subd. 2b). This legislation required the revision of the state's academic standards in mathematics in the 2006–2007 school year. The legislation also required that beginning in the 2013–2014 school year, state mathematics tests given in grade 11 align with the revised 2007 academic standards in mathematics. The revision to the standards was significant enough that a new series of the MCA assessments was necessary. Thus, the Mathematics MCA-III are aligned with the 2007 *Minnesota K–12 Academic Standards in Mathematics*.

[1] At the time of publication, the federal government was reviewing ESEA for changes and reauthorization. Information in this document is up-to-date based on time of publication. If changes are made to ESEA that affect this document, then this document will be revised accordingly in a timely manner.

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MCA-Modified

The Mathematics MCA-Modified (MCA-M) is an alternate assessment based on modified achievement standards. It is designed for a small group of students whose disability has precluded them from achieving grade-level proficiency but who do not qualify to take Minnesota's other alternate assessment, the Minnesota Test of Academic Skills (MTAS). The MCA-Modified differs from the MCA-III in a few key ways:

- The student must have an Individual Education Plan (IEP). The IEP team is responsible for determining, on an annual basis, how a student with a disability will participate in statewide testing. This decision-making process must start with a consideration of the general education assessment.

- The MCA-Modified may only be administered to a student who currently receives special education services, though participation in the administration is not limited to any particular disability category.

- Students must meet all eligibility requirements for the MCA-Modified before it is selected by the IEP team. Eligibility requirements for the MCA-Modified are:

    1. The student demonstrates persistent low performance as defined by performance at the lowest achievement level on the MCA (Does Not Meet the Standards) for the past two years.
       OR
       The student meets or exceeds the standards on the MTAS and the IEP team determines that the student is most appropriately assessed with the MCA-M.

    2. The student has access to instruction on grade-level content standards.

    3. The student has an IEP based on grade-level content standards in the content area(s) being assessed by MCA-M.

DRAFT

4. The IEP team determines that the student is highly unlikely to achieve proficiency on the grade-level content standards within the year the test is administered, even with specially designed instruction.

    a. Objective and valid data from multiple measures should be collected over time to confirm that the student is not likely to achieve proficiency on grade-level content standards within the year. Examples of objective and valid measures include state assessments, district-wide assessments, curriculum-based measures and other repeated measures of progress over time.

    b. Appropriate accommodations, such as assistive technology, are provided as needed on evaluations of classroom performance, and the student's accommodation needs are carefully considered before the IEP team makes a determination that the student is not likely to achieve proficiency on grade-level content standards.

Purpose and Overview of the Test Specifications

The primary purpose of test specifications is to help test developers build a test that is consistent over time. The *DRAFT Mathematics Test Specifications for MCA-III, Grade 11, and MCA-Modified, Grade 11* are also meant to serve as a source of information about the test design for teachers and the general public. Test specifications do not indicate what should be taught; the Minnesota academic standards do. Test specifications do not indicate how students should be taught; the classroom teacher does. Test specifications indicate which strands, standards and benchmarks will be assessed on the test and in what proportions. In addition, test specifications provide the types of items to be included, number of items and distribution of cognitive levels. Test specifications also clarify, define and/or limit how test items will be written.

As with any test, the MCA-III and MCA-Modified assess a sampling of student knowledge and do not test every standard or benchmark. There are standards and benchmarks that cannot be assessed with a standardized test. That does not mean that these skills should not be taught or assessed. Teachers need to instruct and assess

their students on all of the academic standards. Standards and benchmarks that are not assessed on the MCA-III are indicated in this document with the phrase “Not assessed on the MCA-III or MCA-Modified.” In addition, not all assessable benchmarks will be included on every assessment and some benchmarks are embedded within the assessment of other benchmarks.

Panels consisting of members of the Minnesota Academic Standards Committee and classroom teachers were convened to develop the Mathematics MCA-III Test Specifications. Many of the classroom teachers were recommended to the Minnesota Department of Education (MDE) by various education organizations, school districts and other stakeholder groups. For the MCA-Modified, recommendations proposed in focus groups conducted with special educators are incorporated in these specifications.

Item Specifications

Item specifications are provided for each assessed benchmark. The item specifications provide restrictions of numbers, notation, scales, context and item limitations/requirements. The item specifications also list symbols and vocabulary that may be used in items. This list is cumulative in nature. For example, symbols and vocabulary listed at grades 3 through 8 are eligible for use in grade 11. So, symbols and vocabulary listed at grades 3 and 4 are eligible for use in MCA-Modified in grades 5 through 8 and 11.

Item Specification Considerations

There are broad item-development issues addressed during the development of test items. Each of the following issues is considered for all of the items developed for the Mathematics MCA-III and Mathematics MCA-Modified.

1. Each item will be written to measure primarily one benchmark; however, other benchmarks may also be reflected in the item content.
2. Items will be appropriate for students in terms of grade-level difficulty, expected knowledge of grade-level mathematical vocabulary, life experiences and reading level.
3. At a given grade, items will range in difficulty from easy to challenging for the intended population.

DRAFT

4. Items will not disadvantage or disrespect any segment of the population with regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability or geographic region.
5. Items will be written to meet benchmark calculator requirements as specified in the academic standards and/or test specifications.
6. Each item will be written to clearly and unambiguously elicit the desired response.
7. For the Mathematics MCA-III, a reference sheet of appropriate formulas and conversions is provided to students in grades 5–11 for use during testing. For the Mathematics MCA-Modified, appropriate formulas and conversions are provided to students with items in addition to the formula sheet.
8. Items will be written according to the MDE *Guidelines for Test Construction*.
9. Advisory Panels will review items as specified in the MDE *Vendor Guide to Advisory Panels*.
10. Items will be written using principles of Universal Design (see the linguistic modification report from the U.S. Department of Education LEP Partnership[2]). These principles include the following:
    a. Use active voice rather than passive voice.
    b. Avoid negation.
    c. Avoid proper nouns.
    d. Avoid using general language terms that have a special meaning in math contexts.
    e. Reduce written context and be as universal as possible.
    f. To the extent possible, write sentences that are simple and in standard word order.
11. A read-aloud of items is provided via built-in audio in the online assessment.

The MCA-Modified assesses the same grade-level standards as the MCA-III, but the achievement expectations are less rigorous than those on the MCA-III. The same content is covered in the MCA-Modified but with less difficult questions. To meet the goal of providing a test that is accessible yet challenging for the population of students whose disability has prevented them from attaining grade-level proficiency, several design modifications have been made. In addition to the issues outlined and

[2] Abedi, J. & Sato, E. (2008). *Linguistic modification. Part I: Language factors in the assessment of English language learners: The theory and principles underlying the linguistic modification approach; Part II: A guide to linguistic modification: Increasing English language learner access to academic content*. Washington, DC: U.S. Department of Education: LEP Partnership. Available online at http://www.ncela.gwu.edu/files/uploads/11/abedi_sato.pdf

referenced above, the following guidelines should also be used for the MCA-Modified.

1. Items will be written using language simplification principles.
    a. Use high-frequency, familiar vocabulary and short word lengths.
    b. Use short, syntactically non-complex sentences in subject-verb-object order.
    c. Use simple, common verb tenses/moods (infinitive, present indicative, past, simple future); present tense is preferred. Use past participles as adjectives.
    d. Limit use of pronouns; ensure that referents are clear.
    e. Avoid idioms and colloquialisms.
    f. Avoid unnecessary words with multiple meanings.
    g. Avoid long noun and prepositional phrases.
2. Page and item layout will focus on a simplified design.
    a. Increase white space in pages and screens.
    b. Use a single-column format when appropriate.
    c. Stack sentences in stimuli.
    d. Increase size of graphics.
    e. Use simple graphics.
    f. Use uncomplicated art to support item context and meaning.
    g. Limit scrolling in computer-delivered items and two-page layouts in paper forms.
3. The number of operational items within a form are reduced from that used by MCA-III while maintaining the proportion of content coverage across strands, and standards.
4. All items are three-option, multiple-choice questions.
5. Key words are presented in boldface in some items to help students identify the main task to be completed in the item.

Cognitive Complexity

Cognitive complexity refers to the cognitive demand associated with an item. The level of cognitive demand focuses on the type and level of thinking and reasoning required of

DRAFT

the student on a particular item. MCA-III and MCA-Modified levels of cognitive complexity are based on Norman L. Webb's Depth of Knowledge[3] levels.

A Level 1 (recall) item requires the recall of information such as a fact, definition, term or simple procedure, as well as performing a simple algorithm or applying a formula. A well-defined and straight algorithmic procedure is considered to be at this level. A Level 1 item specifies the operation or method of solution and the student is required to carry it out.

A Level 2 (skill/concept) item calls for the engagement of some mental processing beyond a habitual response, with students required to make some decisions as to how to approach a problem or activity. Interpreting information from a simple graph and requiring reading information from the graph is a Level 2. An item that requires students to choose the operation or method of solution and then solve the problem is a Level 2. Level 2 items are often similar to examples used in textbooks.

Level 3 (strategic thinking) items require students to reason, plan or use evidence to solve the problem. In most instances, requiring students to explain their thinking is a Level 3. A Level 3 item may be solved using routine skills but the student is not cued or prompted as to which skills to use.

Level 4 (extended thinking) items require complex reasoning, planning, developing and thinking, most likely over an extended period of time. Level 4 items are best assessed in the classroom, where the constraints of standardized testing are not a factor.

Using these cognitive complexity levels to categorize items ensures that the complexity of the test items matches the complexity of the content domain assessed. Based on the benchmarks included in the Mathematics MCA-III and Mathematics MCA-Modified, Table 1 indicates the target proportion of test items at each cognitive level included in each test.

[3] Webb, N. L. *Alignment of science and mathematics standards and assessments in four states* (Research Monograph No. 18). Madison: University of Wisconsin – Madison, National Institute for Science Education, 1999.

TABLE 1. Cognitive Level Target Minimum Distribution of Items in Mathematics for the MCA and MCA-Modified

Grades	Level 1	Level 2	Level 3
11	20%	30%	5%

Calculator Designation

For Grade 11, there are no non-calculator benchmarks. A calculator will be available for student use on all items of the Grade 11 MCA-III and MCA-Modified.

Test Design by Grade Level

A variety of item types will be used on the Mathematics MCA-III, including multiple-choice (MC) items and technology-enhanced (TE) items. Technology-enhanced items may consist of the following types of responses: type-in (student will type numerical answers in a box), graphing (student will plot data to complete various mathematical displays), drag-and-drop (students will formulate, rather than select, a response using drag-and-drop response options) and hot-spot (students will select multiple correct responses or will mark locations on mathematical graphics and displays).

Table 2 indicates the number of operational items appearing on the paper and online form of the Mathematics MCA-III, as well as the range of items within a test form aligned to each strand. Table 3 indicates the same information for the MCA-Modified.

TABLE 2. Range of Items per Strand for Online and Paper Mathematics MCA-III Grade 11

Grade	Number of Operational Items	Algebra	Geometry & Measurement	Data Analysis & Probability
11	56	25–29	16–18	10–15

TABLE 3. Range of Items per Strand for Mathematics MCA-Modified, Grade 11

Grade	Number of Operational Items	Algebra	Geometry & Measurement	Data Analysis & Probability
11	40	17–21	11–13	7–11

DRAFT

Table 4 indicates the type and number of items for the online Mathematics MCA. Table 5 indicates the type and number of items for the paper Mathematics MCA. As stated previously in this document, the MCA-Modified will use only three-option multiple-choice (MC) items.

TABLE 4. Type and Number of Items for Online Mathematics MCA-III Grade 11

Grade	MC Items (1 point)	Technology-Enhanced Items (1 point)	Total Items/Points
11	48–54	2–8	56

TABLE 5. Type and Number of Items for Paper Mathematics MCA-III Grade 11

Grade	MC Items (1 point)	Fill-In/Gridded Response Items (1 point)	Total Items/Points
11	50–54	2–6	56

The minimum and maximum numbers of operational items by standard for the online and paper Mathematics MCA-III and the MCA-Modified are shown in Tables 6A and 6B.

TABLE 6A. Online and Paper MCA-III Grade 11 Minimum and Maximum Item Counts by Standard

Strand	Range of Items Per Strand	Standard	Number of Benchmarks per Standard	Range of Items Per Standard
2—Algebra	25–29	9.2.1	9	5–7
		9.2.2	6	6–12
		9.2.3	7	4–7
		9.2.4	8	4–8
3—Geometry & Measurement	16–18	9.3.1	5	3–5
		9.3.2	5	0–2
		9.3.3	8	4–8
		9.3.4	7	5–7
4—Data Analysis & Probability	10–15	9.4.1	4	3–6
		9.4.2	3	1–2
		9.4.3	9	5–8

DRAFT

TABLE 6B. Online MCA-Modified Grade 11 Minimum and Maximum Item Counts by Standard

Strand	Range of Items Per Strand	Standard	Number of Benchmarks per Standard	Range of Items Per Standard
2—Algebra	17–21	9.2.1	9	4–6
		9.2.2	6	5–8
		9.2.3	7	3–6
		9.2.4	8	2–7
3—Geometry & Measurement	11–13	9.3.1	5	2–4
		9.3.2	5	0–1
		9.3.3	8	3–7
		9.3.4	7	4–6
4—Data Analysis & Probability	7–11	9.4.1	4	2–5
		9.4.2	3	0–2
		9.4.3	9	4–7

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A Guide to Reading the Test Specifications

Standard
The second level of strand organization

Strand
The general category of content organization

Percent Range by Strand
The percentage of items that test this strand for this grade, listed by test

Grade 11

Strand 3—Geometry & Measurement (28–32%)

Standard 9.3.1: Calculate measurements of plane and solid geometric figures; know that physical measurements depend on the choice of a unit and that they are approximations.

Benchmark
The specific knowledge or skills that students should acquire by the end of that grade level

Item Code
9. Grade (represents grades 9-11)
3. Strand
1. Standard
3. Benchmark

Item Specifications
The clarification, definition or restriction of items assessing this benchmark

Benchmarks

9.3.1.1
Determine the surface area and volume of pyramids, cones and spheres. Use measuring devices or formulas as appropriate.
Item Specifications
Vocabulary allowed in items: sphere and vocabulary given at previous grades

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9.3.1.2
Compose and decompose two- and three-dimensional figures; use decomposition to determine the perimeter, area, surface area and volume of various figures.
Item Specifications
Vocabulary allowed in items: regular polygon, sphere, compose, decompose and vocabulary given at previous grades

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9.3.1.3
Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems.
Item Specifications
Vocabulary allowed in items: vocabulary given at previous grades

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9.3.1.4
Understand and apply the fact that the effect of a scale factor k on length, area and volume is to multiply each by k, k^2 and k^3, respectively.
Item Specifications
Vocabulary allowed in items: scale factor, magnitude and vocabulary given at previous grades

DRAFT

An Explanation of Terms Related to the Grade-Level Tables

Strand: This is the most general categorization of content in the Minnesota Academic Standards.

Standard: Standards describe the expectations in mathematics that all students must satisfy to meet state requirements for credit.

Benchmark: The purpose of benchmarks is to provide details about "the academic knowledge and skills that schools must offer and students must achieve to satisfactorily complete" the standards (Minn. Stat. § 120B.023 (2006)). Benchmarks are intended to "inform and guide parents, teachers, school districts and other interested persons and for use in developing tests consistent with the benchmarks" (Minn. Stat. § 120B.023 (2006)). Each standard is divided into several benchmarks.

Item Code: Test developers use this code to identify the strand, standard and benchmark to which a test item is aligned.

Item Specifications: These statements provide more specific clarifications, definitions or restrictions for the benchmark as it is assessed on the MCA.

Percent Range by Strand: This range is the possible percentage of items that will be on the operational form from a specific strand.

DRAFT

Mathematics MCA-III Test Specifications Grade 11 Level Tables

Strand 2—Algebra (45–52%)

Standard 9.2.1: Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.

Benchmarks

9.2.1.1

Understand the definition of a function. Use functional notation and evaluate a function at a given point in its domain.

Item Specifications

- Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

---

9.2.1.2

Distinguish between functions and other relations defined symbolically, graphically or in tabular form.

Item Specifications

- Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

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9.2.1.3

Find the domain of a function defined symbolically, graphically or in a real-world context.

Item Specifications

- Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

---

9.2.1.4

Obtain information and draw conclusions from graphs of functions and other relations.

Item Specifications

- Vocabulary allowed in items: relation, domain, range and vocabulary given at previous grades

---

9.2.1.5

Identify the vertex, line of symmetry and intercepts of the parabola corresponding to a quadratic function, using symbolic and graphical methods, when the function is expressed in the form $f(x) = ax^2 + bx + c$, in the form $f(x) = a(x - h)^2 + k$, or in factored form.

Item Specifications

- Vocabulary allowed in items: line of symmetry, parabola, quadratic, vertex and vocabulary given at previous grades.

DRAFT

9.2.1.6
Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.
Item Specifications
- Vocabulary allowed in items: maximum, minimum, interval, zeros and vocabulary given at previous grades

---

9.2.1.7
Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods.
Item Specifications
- Vocabulary allowed in items: asymptote and vocabulary given at previous grades

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9.2.1.8
Make qualitative statements about the rate of change of a function, based on its graph or table of values.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

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9.2.1.9
Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

Standard 9.2.2: Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.

Benchmarks
9.2.2.1
Represent and solve problems in various contexts using linear and quadratic functions.
Item Specifications
- Vocabulary allowed in items: quadratic and vocabulary given at previous grades

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9.2.2.2
Represent and solve problems in various contexts using exponential functions, such as investment growth, depreciation and population growth.
Item Specifications
- Vocabulary allowed in items: growth factor, decay, exponential and vocabulary given at previous grades

DRAFT

9.2.2.3
Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables and symbolic representations. Know how to use graphing technology to graph these functions.
Item Specifications
- Items do not require the use of graphing technology
- Vocabulary allowed in items: quadratic, exponential and vocabulary given at previous grades

---

9.2.2.4
Express the terms in a geometric sequence recursively and by giving an explicit (closed form) formula, and express the partial sums of a geometric series recursively.
Item Specifications
- Vocabulary allowed in items: recursive, geometric series, common ratio and vocabulary given at previous grades

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9.2.2.5
Recognize and solve problems that can be modeled using finite geometric sequences and series, such as home mortgage and other compound interest examples. Know how to use spreadsheets and calculators to explore geometric sequences and series in various contexts.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

---

9.2.2.6
Sketch the graphs of common non-linear functions such as $f(x)=\sqrt{x}$, $f(x)=|x|$, $f(x)=1/x$, $f(x)=x^3$ and translations of these functions, such as $f(x)=\sqrt{(x-2)} + 4$. Know how to use graphing technology to graph these functions.
Item Specifications
- Items do not require the use of graphing technology
- Vocabulary allowed in items: vocabulary given at previous grades

Standard 9.2.3: Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.

Benchmarks
9.2.3.1
Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified points in their domains.
Item Specifications
- Vocabulary allowed in items: polynomial and vocabulary given at previous grades

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9.2.3.2
Add, subtract and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.
Item Specifications
- Vocabulary allowed in items: polynomial, degree of a polynomial and vocabulary given at previous grades

DRAFT

9.2.3.3
Factor common monomial factors from polynomials, factor quadratic polynomials, and factor the difference of two squares.
Item Specifications
- Vocabulary allowed in items: polynomial, monomial and vocabulary given at previous grades

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9.2.3.4
Add, subtract, multiply, divide and simplify algebraic fractions.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

---

9.2.3.5
Check whether a given complex number is a solution of a quadratic equation by substituting it for the variable and evaluating the expression, using arithmetic with complex numbers.
Item Specifications
- Vocabulary allowed in items: complex number and vocabulary given at previous grades

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9.2.3.6
Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions, including those involving n^{th} roots.
Item Specifications
- Vocabulary allowed in items: n^{th} root and vocabulary given at previous grades

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9.2.3.7
Justify steps in generating equivalent expressions by identifying the properties used. Use substitution to check the equality of expressions for some particular values of the variables; recognize that checking with substitution does not guarantee equality of expressions for all values of the variables.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

Standard 9.2.4: Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential and nth root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.

Benchmarks
9.2.4.1
Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities.
Item Specifications
- Items do not require the use of graphing technology
- Vocabulary allowed in items: quadratic, n^{th} root and vocabulary given at previous grades

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9.2.4.2
Represent relationships in various contexts using equations involving exponential functions; solve these equations graphically or numerically. Know how to use calculators, graphing utilities or other technology to solve these equations.
Item Specifications
- Items do not require the use of graphing technology
- Vocabulary allowed in items: exponential and vocabulary given at previous grades

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9.2.4.3
Recognize that to solve certain equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers, and from real numbers to complex numbers. In particular, non-real complex numbers are needed to solve some quadratic equations with real coefficients.
Item Specifications
- Vocabulary allowed in items: quadratic, complex, non-real and vocabulary given at previous grades

---

9.2.4.4
Represent relationships in various contexts using systems of linear inequalities; solve them graphically. Indicate which parts of the boundary are included in and excluded from the solution set using solid and dotted lines.
Item Specifications
- Vocabulary allowed in items: boundary and vocabulary given at previous grades

---

9.2.4.5
Solve linear programming problems in two variables using graphical methods.
Item Specifications
- Vocabulary allowed in items: constraint, boundary, feasible region and vocabulary given at previous grades

---

9.2.4.6
Represent relationships in various contexts using absolute value inequalities in two variables; solve them graphically.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

---

9.2.4.7
Solve equations that contain radical expressions. Recognize that extraneous solutions may arise when using symbolic methods.
Item Specifications
- Vocabulary allowed in items: extraneous and vocabulary given at previous grades

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9.2.4.8
Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

DRAFT

Strand 3—Geometry & Measurement (28–32%)

Standard 9.3.1: Calculate measurements of plane and solid geometric figures; know that physical measurements depend on the choice of a unit and that they are approximations.

Benchmarks

9.3.1.1

Determine the surface area and volume of pyramids, cones and spheres. Use measuring devices or formulas as appropriate.

Item Specifications

- Vocabulary allowed in items: sphere and vocabulary given at previous grades

---

9.3.1.2

Compose and decompose two- and three-dimensional figures; use decomposition to determine the perimeter, area, surface area and volume of various figures.

Item Specifications

- Vocabulary allowed in items: regular polygon, sphere, compose, decompose and vocabulary given at previous grades

---

9.3.1.3

Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems.

Item Specifications

- Vocabulary allowed in items: vocabulary given at previous grades

---

9.3.1.4

Understand and apply the fact that the effect of a scale factor k on length, area and volume is to multiply each by k, k^2 and k^3, respectively.

Item Specifications

- Vocabulary allowed in items: scale factor, magnitude and vocabulary given at previous grades

---

9.3.1.5

Make reasonable estimates and judgments about the accuracy of values resulting from calculations involving measurements.

Item Specifications

- Assessed within 9.3.1.1 through 9.3.1.4

Standard 9.3.2: Construct logical arguments, based on axioms, definitions and theorems, to prove theorems and other results in geometry.

Benchmarks

9.3.2.1

Understand the roles of axioms, definitions, undefined terms and theorems in logical arguments.

Item Specifications

- Assessed within 9.3.2.2 and 9.3.2.4

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9.3.2.2
Accurately interpret and use words and phrases such as "if…then," "if and only if," "all," and "not." Recognize the logical relationships between an "if…then" statement and its inverse, converse and contrapositive.
Item Specifications
- Allowable notation: $p \rightarrow q$ (if p then q)
- Vocabulary allowed in items: inverse, converse, contrapositive, negation and vocabulary given at previous grades

---

9.3.2.3
Assess the validity of a logical argument and give counterexamples to disprove a statement.
Item Specifications
- Assessed within 9.3.2.4

9.3.2.4
Construct logical arguments and write proofs of theorems and other results in geometry, including proofs by contradiction. Express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts or illustrations.
Item Specifications
- Allowable notation: CPCTC (Corresponding Parts of Congruent Triangles are Congruent), CPCFC (Corresponding Parts of Congruent Figures are Congruent), SAS (Side-Angle-Side), SSS (Side-Side-Side), ASA (Angle-Side-Angle)
- Vocabulary allowed in items: contradiction and vocabulary given at previous grades

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9.3.2.5
Use technology tools to examine theorems, make and test conjectures, perform constructions and develop mathematical reasoning skills in multi-step problems. The tools may include compass and straight edge, dynamic geometry software, design software or Internet applets.
Item Specifications
- Vocabulary allowed in items: angle bisector, perpendicular bisector, midpoint of a segment and vocabulary given at previous grades

Standard 9.3.3: Know and apply properties of geometric figures to solve real-world and mathematical problems and to logically justify results in geometry.

Benchmarks
9.3.3.1
Know and apply properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve problems and logically justify results.
Item Specifications
- Allowable notation: $\perp$ (perpendicular), $\parallel$ (parallel)
- Vocabulary allowed in items: transversal, interior, exterior, corresponding, alternate and vocabulary given at previous grades

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9.3.3.2
Know and apply properties of angles, including corresponding, exterior, interior, vertical, complementary and supplementary angles, to solve problems and logically justify results.
Item Specifications
- Vocabulary allowed in items: transversal, interior, exterior, corresponding, alternate, vertical and vocabulary given at previous grades

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9.3.3.3
Know and apply properties of equilateral, isosceles and scalene triangles to solve problems and logically justify results.
Item Specifications
- Vocabulary allowed in items: equilateral, isosceles, scalene and vocabulary given at previous grades

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9.3.3.4
Apply the Pythagorean Theorem and its converse to solve problems and logically justify results.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

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9.3.3.5
Know and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems and logically justify results.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

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9.3.3.6
Know and apply properties of congruent and similar figures to solve problems and logically justify results.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

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9.3.3.7
Use properties of polygons—including quadrilaterals and regular polygons—to define them, classify them, solve problems and logically justify results.
Item Specifications
- Vocabulary allowed in items: regular polygon, isosceles and vocabulary given at previous grades

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9.3.3.8
Know and apply properties of a circle to solve problems and logically justify results.
Item Specifications
- Vocabulary allowed in items: arc, central angle, inscribed, circumscribed, tangent, chord and vocabulary given at previous grades

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Standard 9.3.4: Solve real-world and mathematical geometric problems using algebraic methods.

Benchmarks

9.3.4.1

Understand how the properties of similar right triangles allow the trigonometric ratios to be defined, and determine the sine, cosine and tangent of an acute angle in a right triangle.

Item Specifications

- Items do not include context.
- Vocabulary allowed in items: trigonometric ratios, sine, cosine, tangent and vocabulary given at previous grades

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9.3.4.2

Apply the trigonometric ratios sine, cosine and tangent to solve problems, such as determining lengths and areas in right triangles and in figures that can be decomposed into right triangles. Know how to use calculators, tables or other technology to evaluate trigonometric ratios.

Item Specifications

- Vocabulary allowed in items: trigonometric ratios, sine, cosine, tangent and vocabulary given at previous grades

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9.3.4.3

Use calculators, tables or other technologies in connection with the trigonometric ratios to find angle measures in right triangles in various contexts.

Item Specifications

- Assessed within 9.3.4.1 and 9.3.4.2

9.3.4.4

Use coordinate geometry to represent and analyze line segments and polygons, including determining lengths, midpoints and slopes of line segments.

Item Specifications

- Vocabulary allowed in items: midpoint and vocabulary given at previous grades

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9.3.4.5

Know the equation for the graph of a circle with radius r and center (h, k), $(x - h)^2 + (y - k)^2 = r^2$, and justify this equation using the Pythagorean Theorem and properties of translations.

Item Specifications

- Vocabulary allowed in items: vocabulary given at previous grades

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9.3.4.6

Use numeric, graphic and symbolic representations of transformations in two dimensions, such as reflections, translations, scale changes and rotations about the origin by multiples of 90˚, to solve problems involving figures on a coordinate grid.

Item Specifications

Allowable notation: P' (P prime)

- Vocabulary allowed in items: pre-image, image, isometry and vocabulary given at previous grades

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9.3.4.7
Use algebra to solve geometric problems unrelated to coordinate geometry, such as solving for an unknown length in a figure involving similar triangles, or using the Pythagorean Theorem to obtain a quadratic equation for a length in a geometric figure.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

Strand 4—Data Analysis & Probability (18–26%)

Standard 9.4.1: Display and analyze data; use various measures associated with data to draw conclusions, identify trends and describe relationships.

Benchmarks
9.4.1.1
Describe a data set using data displays, including box-and-whisker plots; describe and compare data sets using summary statistics, including measures of center, location and spread. Measures of center and location include mean, median, quartile and percentile. Measures of spread include standard deviation, range and inter-quartile range. Know how to use calculators, spreadsheets or other technology to display data and calculate summary statistics.
Item Specifications
- Vocabulary allowed in items: box-and-whisker plot, quartile, percentile, inter-quartile range, standard deviation, central tendency and vocabulary given at previous grades

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9.4.1.2
Analyze the effects on summary statistics of changes in data sets.
Item Specifications
- Vocabulary allowed in items: quartile, percentile, inter-quartile range, standard deviation, central tendency and vocabulary given at previous grades

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9.4.1.3
Use scatterplots to analyze patterns and describe relationships between two variables. Using technology, determine regression lines (line of best fit) and correlation coefficients; use regression lines to make predictions and correlation coefficients to assess the reliability of those predictions.
Item Specifications
- Vocabulary allowed in items: regression line, correlation coefficient and vocabulary given at previous grades

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9.4.1.4
Use the mean and standard deviation of a data set to fit it to a normal distribution (bell-shaped curve) and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.
Item Specifications
- Allowable notation: $\overline{x}$ (mean of a sample)
- Vocabulary allowed in items: standard deviation, normal distribution, normal curve and vocabulary given at previous grades

Standard 9.4.2: Explain the uses of data and statistical thinking to draw inferences, make predictions and justify conclusions.

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Benchmarks
9.4.2.1
Evaluate reports based on data published in the media by identifying the source of the data, the design of the study, and the way the data are analyzed and displayed. Show how graphs and data can be distorted to support different points of view. Know how to use spreadsheet tables and graphs or graphing technology to recognize and analyze distortions in data displays.
Item Specifications
- Not assessed on the MCA-III

9.4.2.2
Identify and explain misleading uses of data; recognize when arguments based on data confuse correlation and causation.
Item Specifications
- Vocabulary allowed in items: causation and vocabulary given at previous grades

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9.4.2.3
Design simple experiments and explain the impact of sampling methods, bias and the phrasing of questions asked during data collection.
Item Specifications
- Items do not require students to design experiments.
- Vocabulary allowed in items: vocabulary given at previous grades

Standard 9.4.3: Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.

Benchmarks
9.4.3.1
Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

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9.4.3.2
Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.
Item Specifications
- Vocabulary allowed in items: simulation and vocabulary given at previous grades

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9.4.3.3
Understand that the Law of Large Numbers expresses a relationship between the probabilities in a probability model and the experimental probabilities found by performing simulations or experiments involving the model.
Item Specifications
- Vocabulary allowed in items: simulation and vocabulary given at previous grades

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9.4.3.4
Use random numbers generated by a calculator or a spreadsheet, or taken from a table, to perform probability simulations and to introduce fairness into decision making.
Item Specifications
- Items do not require students to generate random numbers
- Vocabulary allowed in items: simulation and vocabulary given at previous grades

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9.4.3.5
Apply probability concepts such as intersections, unions and complements of events, and conditional probability and independence, to calculate probabilities and solve problems.
Item Specifications
- Vocabulary allowed in items: intersections, unions, complements of events, conditional and vocabulary given at previous grades

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9.4.3.6
Describe the concepts of intersections, unions and complements using Venn diagrams. Understand the relationships between these concepts and the words AND, OR, NOT, as used in computerized searches and spreadsheets.
Item Specifications
- Allowable notation: ∪ (union), ∩ (intersection)
- Vocabulary allowed in items: intersections, unions, complements and vocabulary given at previous grades

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9.4.3.7
Understand and use simple probability formulas involving intersections, unions and complements of events.
Item Specifications
- Vocabulary allowed in items: intersections, unions and complements of events and vocabulary given at previous grades

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9.4.3.8
Apply probability concepts to real-world situations to make informed decisions.
Item Specifications
- Vocabulary allowed in items: vocabulary given at previous grades

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9.4.3.9
Use the relationship between conditional probabilities and relative frequencies in contingency tables.
Item Specifications
- Vocabulary allowed in items: conditional and vocabulary given at previous grades

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